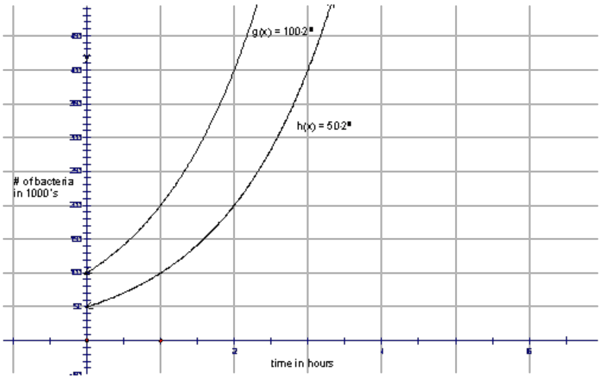
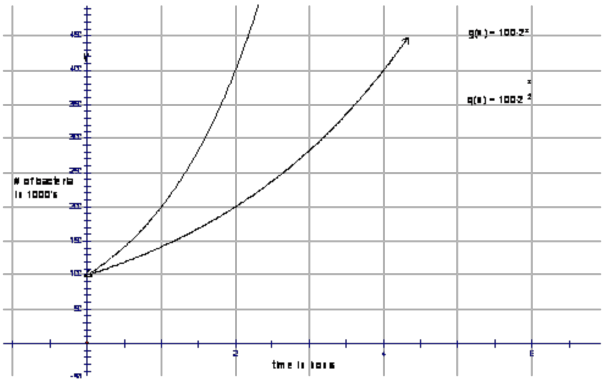
6.7 Exponential Growth

Bacterial Growth





Exponential Growth: Comparing Functions

Compare the graphs on each grid, list and explain their differences and similarities.

$g(x)=100(2)^x$ $h(x)=50(2)^x$

Similarities
- both exponential growth
- base of 2 - has doubling time
- both have a doubling time of 1 hour

Differences
- initial values are different

$g(x)=100(2)^x$ $f(x)=100(2)^{x/2}$

Similarities
- both exponential growth
- base of 2 - has doubling time
- same initial value

Differences
- doubling time is different
- $g(x)=100(2)^x$ doubles in 1 hour
- $g(x)=100(2)^{x/2}$ doubles in 2 hours

Formula: $P(n)=P_0(1+r)^n$

➡ P(n): final amount (number)
r: growth rate

P_0: initial amount (number)
n: number of growth periods

When the base (1+r) is greater than 1: growth

Ex 1:

According to the 1991 census, the region of Niagara, Ontario, had 364 552 residents. For the next few years, the population grew at a rate of 2.2% per year. For planning purposes, the regional government needs to determine the population of the region in 2010. The algebraic model for this case is: $P(n)=P_0(1+r)^n$

a) What is the initial population, P_0? 364552

b) What is the growth rate r of the population? $\frac{2.2}{100} = 0.022$

c) How many growth periods are there? 2010−1991 = 19

d) Write the algebraic model for this situation.

$P(n) = 364552(1+0.022)^n$

e) Use the model to determine the population in 2010.

$= 364552(1.022)^n$

$= 364552(1+0.022)^{19}$

$= 551221.9529$

∴ the population will be 551222 in 2010

Real World Applications

1. You can calculate the amount of money you have in your investment account after a given number of years using the formula $A = P(1+i)^n$ where A is the amount in $ you will have, P is the principal (initial amount of money invested), i is the interest rate per year and n is the number of years.

a) Complete the table of values for your $5000 invested at 4.5%/a.

n	A
1	5225
2	5460.13
3	5705.83
4	5962.59
5	6230.91

b) When will you have $ 5705 ? 3

c) When will you have $6095 ? 4.5

d) This is an example of exponential growth

$$A = 5000(1+0.045)^n$$

$$\frac{6095}{5000} = \frac{5000(1.045)^n}{5000}$$

$$1.219 = (1.045)^n$$

$$\frac{\log 1.219}{\log 1.045}$$

$$n = 4.5$$

Ex 2: (p 430 #8)

Mari invests $2000 in a bond that pays 6% per year.

P r

a) Write an equation that models the growth of her investment.

$$A = 2000(1+0.06)^n$$

b) How much money does she have if she cashes the bond at the end of the 4th year?

$$= 2000(1+0.06)^4$$

$$= 2524.95$$

$\therefore$ after 4 years she will have $\boxed{\$}$2524.95

c) How much will the bond be worth at the end of the 5th year?

$$= 2000(1+0.06)^5$$

$$= 2676.45$$

$\therefore$ $2676.45

How can you determine the amount earned during the 5th year?

year 5 − year 4 $\therefore$ $151.50 in interest earned.

e) Determine the amount Mari will earn at the end of the 20th and 21st year and the amount made during the 21st year

$A_{(21)} = 6799.13$ $A_{(20)} = 6414.27$

During the 21^{st} year $= 6799.13 - 6414.27$
$= 384.86$

$\therefore$ \$384.86 is made during the 21^{st} year.

f) Compare the money earned during the 5th and 21rst years. What does this tell you about exponential growth?

Money grows faster the longer it is invested

Ex 3: (p 431 #10)

$f(x) = ca^x$

An ant colony triples in number every month. Currently, there are 24000 in the nest.

a) What is the initial population?

24000

b) Write an equation that models the number of ants in the colony, given the number of months.

$f(x) = 24000(3)^x$

c) Use your equation to predict the size of the colony in three months.

$f(x) = 24000(3)^3$

$\therefore$ ______ $= 648000$

negative

d) Use your equation to predict the size of the colony five months ago.

$f(x) = 24000(3)^{-5}$
$= 99$

$\therefore$ ______

Ex 4 A certain strain of yeast cell doubles under certain conditions every 20 minutes. If there were 350 initially, how many cells will there be in 3 hours?

$= 350(2)^{\frac{t}{20}}$

$= 350(2)^{\frac{180}{20}}$

$= 179200$

3 hours = 180 min

$\therefore$ the population is 179200 after 3 hours.

Hmwk:
p 430 # 4, 6, 7, 9, 11